Lesson 6-4: Homework Example #31

#31: Suppose that a cup of soup cooled from 90^{o}C to 60^{o}C in 10 minutes in a room whose temperature was 20^{o}C. Use Newton's Law of Cooling to find (a) how much longer would it take the soup to cool to 35^{o}C, and (b) if the soup had immediately been put into a freezer whose temperature is -15^{o}C instead of being left to stand in the room, how long would it take the soup to cool to 35^{o}C?

$T = T_s + (t_0 - T_s)e^{kt}$ $t_0 = 90^{o}$C $T = T_s + (90 - T_s)e^{kt}$	This is Newton's law of Cooling and the initial temperature of the soup. We can put the initial temperature into the equation.
At t = 10, T = 60^{o}C $T_s = 20^{o}$C	For part (a) we were also given some information.
$T = 20 + (90 - 20)e^{kt}$ $T = 20 + 70e^{kt}$ At t = 10, T = 60^{o}C $60 = 20 + 70e^{10k}$ $40 = 70e^{10k}$ $\frac{4}{7} = e^{10k}$ 10k = ln (4/7) k = (1/10) ln (4/7) $T = 20 + 70e^{(1/10)(\ln(4/7))t}$	The surrounding temperature can be put in right away. Use the other information to find the value of k. Write your equation.
t = ? when T = 35 $35 = 20 + 70e^{(1/10)(\ln(4/7))t}$ $15 = 70e^{(1/10)(\ln(4/7))t}$ $15 = 70e^{(1/10)(\ln(4/7))t}$ (1/10) ln (4/7)t = ln(3/14) $t \approx 27.53\,\text{min}$	Finally, for part (a) we can find the time it will take to cool the soup to 35^{o}C.
$T_s = -15^{o}$C $T = -15 + 105e^{(1/10)(\ln(4/7))t}$ t = ? when T = 35 $35 = -15 + 105e^{(1/10)(\ln(4/7))t}$ $50 = 105e^{(1/10)(\ln(4/7))t}$ (1/10) ln (4/7)t = ln(10/21) $t \approx 13.26\,\text{min}$	For (b) we are going to immediately put the soup into the freezer. We need to know how long it will take this time to lower the temperature. We can use the same value for k as in part (a).